
ECONOMICS

& Sociology

Kalinowski, S. (2015), Price Discount for the Increased Order as a Cooperative Game in Bilateral Monopoly, *Economics and Sociology*, Vol. 8, No 3, pp. 108-118.
DOI: 10.14254/2071-789X.2015/8-3/8

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Received: April, 2015
1st Revision: July, 2015
Accepted: August, 2015

DOI: 10.14254/2071-789X.2015/8-3/8

PRICE DISCOUNT FOR THE INCREASED ORDER AS A COOPERATIVE GAME IN BILATERAL MONOPOLY

ABSTRACT. The bilateral monopoly is the market structure that joins sole producer of the good and the monopolistic distributor of it. There are two possible solutions to the determination of price and quantity traded between buyer and seller. First, non-cooperative one, is the price leadership of the seller producing the Bowley equilibrium. Second is cooperative solution maximizing joint profits with undetermined price. The price level shares the sum of profits between buyer and seller. The article applies the Nash bargaining solution to determine the agreement point in two stage cooperation. The aim of the study is to investigate, what will be the cooperative solution if the buyer and seller achieve the Bowley equilibrium point first and then negotiate cooperative set of the price discount and the quantity traded growth rate. The outcome of the model is the asymmetric division of the maximized joint profit. Thanks to his price leadership, the share of the seller is significantly higher than for the one stage cooperation.

JEL Classification: C71, C78, **Keywords:** bilateral monopoly, Nash bargaining solution, cooperative games, Bowley equilibrium.

Introduction

The discussion on bilateral monopoly was opened by the article by Albert L. Bowley in 1928. He developed the general idea of Wicksell's underlying indetermination of the solution. Bilateral monopoly, in general understanding, is the situation of the constant sum of profits to be shared between buyer and seller in many ways (Bowley, 1928, p. 654). This relationship is a natural subject for cooperative solution drawn either from the negotiation or from one of the bargaining solutions proposed by the game theory. Bowley introduced the assumption that canceled the indetermination of the solution (Bowley, 1928, p. 655): let's grant the seller the price leadership. He establishes the price and the buyer answers with the order that maximizes his profit. The seller knows the demand function for the market on which the buyer operates. Knowing the buyer's best response function the seller chooses the price that maximizes his profit.

The next important achievement in the field of bilateral monopoly was the study by William Fellner which reinterpreted Bowley's three cases for the labor market. Fellner

founded his analysis on the indifference curves of the trade union and the employer. The outcome of the model was the limitations of the wage bargaining range (Fellner, 1947, p. 530). There is also the conclusion that the possible result of the negotiations depend on the willingness of both sides to move toward opponent's proposal. Fellner concluded that "a permanent stalemate develops only if each party overestimates the willingness of the other to move" (Fellner, 1947, p. 531), but on the other hand „if one of the two parties estimates the other party's willingness to move correctly, an agreement will be reached, regardless of whether the estimate of the other party also is correct". Fellner's model emphasizes the non-economic nature of the indeterminateness of the cooperative solution in bilateral monopoly.

Fritz Machlup and Martha Taber indicated that "joint profit maximization itself can be achieved by other means than vertical merger; loose-knit agreements and, of course, direct and free negotiations of quantities and prices may do the job" (Machlup, Taber, 1960, p. 116). Authors paved the path of the research leading to alternative, dynamic solutions to bilateral monopoly. This inspired the series of experimental studies of bilateral monopoly (Fouraker, Siegel, Harnett, 1962; Siegel, Harnett, 1964; Harnett, 1967). Fouraker Siegel and Harnett revealed the dominance of the cooperative joint profit maximization in the experiments with iterated price and quantity bids, especially in the case of complete information about opponents profits. Chatterjee and Samuelson also proved the importance of the information conditions for the probability of negotiation's success. The set of theoretical models led to the conclusion that the more buyer and seller know about the reservation prices of the opponents, the more likely is the agreement (Chatterjee, Samuelson, 1983, p. 849).

Following the idea of Machlup and Taber inspiring to look for alternative bilateral monopoly solutions, Dobbs and Hill proved that state contingent contracts *take-or pay* (buyer has to buy agreed minimum quantity, otherwise he has to pay for it anyway) with non-uniform prices lead to static bilateral monopoly solution (Dobbs, Hill, 1993, p. 486). Another study which introduces factors implying the determinate bilateral monopoly solution is the article by Truett and Truett. The authors are pointing the real life circumstances that may drive the solution towards or backwards the joint profits maximization (Truett, Truett, 1993). Also in the Irmen's article the is the additional assumption that "managers use cost-based percentage margins when pricing their goods, these margins should be determined as equilibrium choices. This paper studies the case of bilateral monopoly and compares the Nash equilibrium in percentage and in absolute mark-ups. We show that percentage mark-ups lead to lower equilibrium prices and higher downstream profits" (Irmen, 1997, p. 179). Dasgupta and Devadoss introduced the utility functions of profits and bargaining power of both sides instead of joint profits to be maximized. The modified multi-period Bowley price leadership model produced the conditions that induced Nash equilibrium at jointly determined points of operation (Dasgupta, Devadoss, 2002, p. 43).

Since 1950, Nash bargaining solution opened new research area as far as the bilateral monopoly is concerned. It pointed the price quantity bid which maximizes joint profits and is acceptable for both sides regarding to status quo point which is the pair of profits in case of no agreement (Nash, 1950a; Nash, 1953). Speaking of game theoretical approach, seller's price leadership producing Bowley point is the of Nash equilibrium in bilateral monopoly (Nash, 1950b).

The following chapters investigate influence of the new factor in bilateral monopoly, two stage cooperation between buyer and seller. First, there is the non-cooperative equilibrium with the price leadership of the seller in the Bowley point. Second stage is the cooperative solution pointing the agreed price discount and the quantity traded growth rate. The purpose of this paper is to find out whether two stage cooperation produces different bargaining solution in comparison to the one stage agreement establishing price and quantity.

1. Presentation of the model

The subject of the analysis is the situation of price discount bargaining between two enterprises S (seller) and B (buyer). Buyer operates on the retail market of the traded good. The demand on this market can be described by the simple linear function:

$$P - bq = a - bq \quad a, b \in R^+, \quad (1)$$

where:

P – retail price of the traded good,

q – quantity sold,

a – maximum price at the $q=0$,

b – the absolute value of the slope of the demand curve.

Buyer purchases the traded good from the seller at a price p . Hence, his profit function is:

$$\pi_b(q, p) = aq - bq^2 - pq - f_b, \quad (2)$$

where:

p – purchase price of the traded good for the buyer,

f_b – fixed cost of the buyer.

Seller produces traded good at the certain variable cost v . Hence, his profit function is:

$$\pi_s(q, p) = pq - vq - f_s, \quad (3)$$

where:

v – unit variable cost of production of the traded good,

f_s – fixed cost of the seller.

Both, buyer and the seller are joined with the particular relationship which is bilateral monopoly with price leadership of the seller. The buyer maximizes his profit buying the quantity equaling the first coordinate of the profit function to zero:

$$\pi'_b(q) = a - 2bq - p = 0, \quad (4)$$

which holds for:

$$q = \frac{a-p}{2b}. \quad (5)$$

Seller, knowing the market profit function, and assuming maximizing behavior of the buyer chooses the price level maximizing his profit:

$$\pi'_s(p) = p - v \frac{a-p}{2b} - f_s' = 0, \quad (6)$$

which holds for:

$$p = \frac{a+v}{2}. \quad (7)$$

According to equation (5), buyer reacts for this transaction price ordering:

$$q = \frac{a-v}{4b}. \quad (8)$$

Price and quantity defined by the equations (7) and (8) set the Bowley equilibrium point for buyer and seller. Any of them is not interested in changing the price and the quantity. This does not mean, that profits of the traders can't be higher.

Let's presume that seller offers price discount (d) in return for the enlargement (e) of the buyer's order. Under such circumstances, profits of both sides can be defined by the formulas:

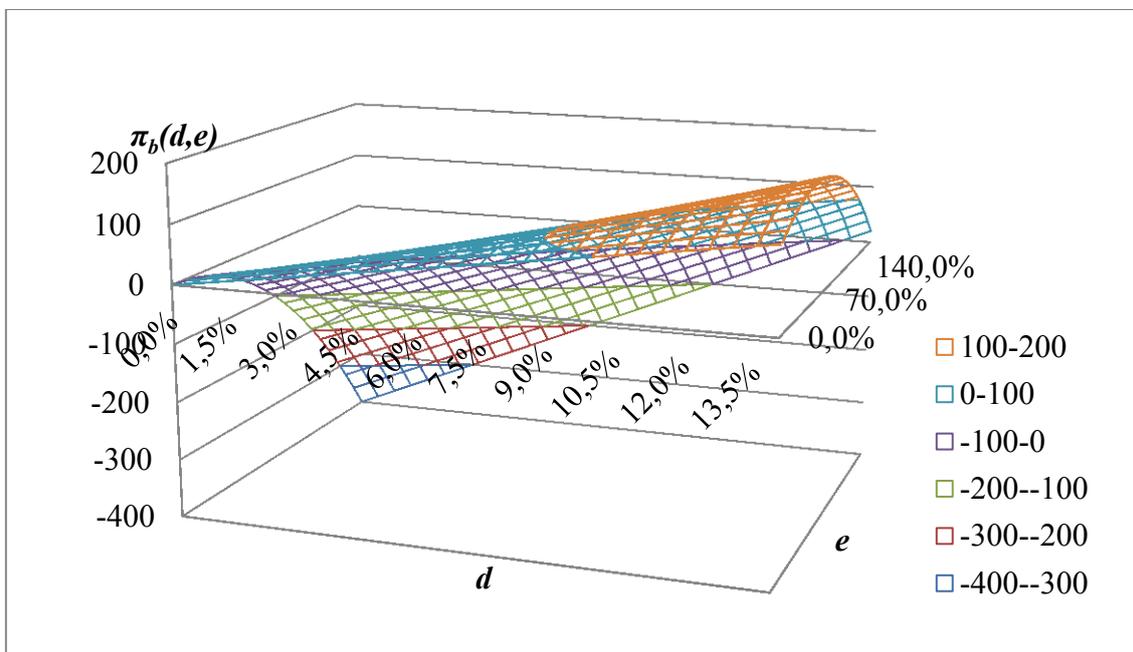
$$\pi_b \ q, p, d, e = aq(1 + e) - bq^2 \ 1 + e^2 - p(1 - d)q(1 + e) - f_b, \quad (9)$$

$$\pi_s \ q, p, d, e = p(1 - d)q(1 + e) - vq(1 + e) - f_s, \quad (10)$$

where:

e – growth rate of the buyer's order,

d – seller's price discount for enlarged purchase.



Graph 1. Profit of the buyer

Profit of the buyer rises as the price discount is increasing. The relationship between buyer's profit and the quantity sales growth is slightly more complex. First, profit is growing with the e increasing. After crossing a threshold point the relationship is negative. Hence, there exists the possibility to calculate optimal quantity growth rate for every price discount (see equation 28).

Seller's profit is rising with the rise of the quantity growth rate. The slope of the profit surface is the highest for zero discount rate. The higher is d , the slower is profit increase for the buyer following the rise of e .

Bearing in mind, that the status quo point is the Bowley equilibrium, the profit functions of both enterprises can be expressed as such:

$$\pi_b \ d, e = \frac{a(a-v)}{4b} (1 + e) - \frac{(a-v)^2}{16b} \ 1 + e^2 - \frac{a^2-v^2}{8b} (1 - d)(1 + e) - f_b, \quad (11)$$

$$\pi_s \ d, e = \frac{a^2-v^2}{8b} (1 - d)(1 + e) - v \frac{a-v}{4b} (1 + e) - f_s, \quad (12)$$

Price discount agreement is possible if new profits are higher than the ones without price discount agreement. This produces the following condition for the buyer:

$$\pi_b(q, p, d, e) > \pi_b(q, p), \tag{13}$$

$$aq(1+e) - bq^2(1+e)^2 - p(1-d)q(1+e) - f_b > aq - bq^2 - pq - f_b, \tag{14}$$

which holds for:

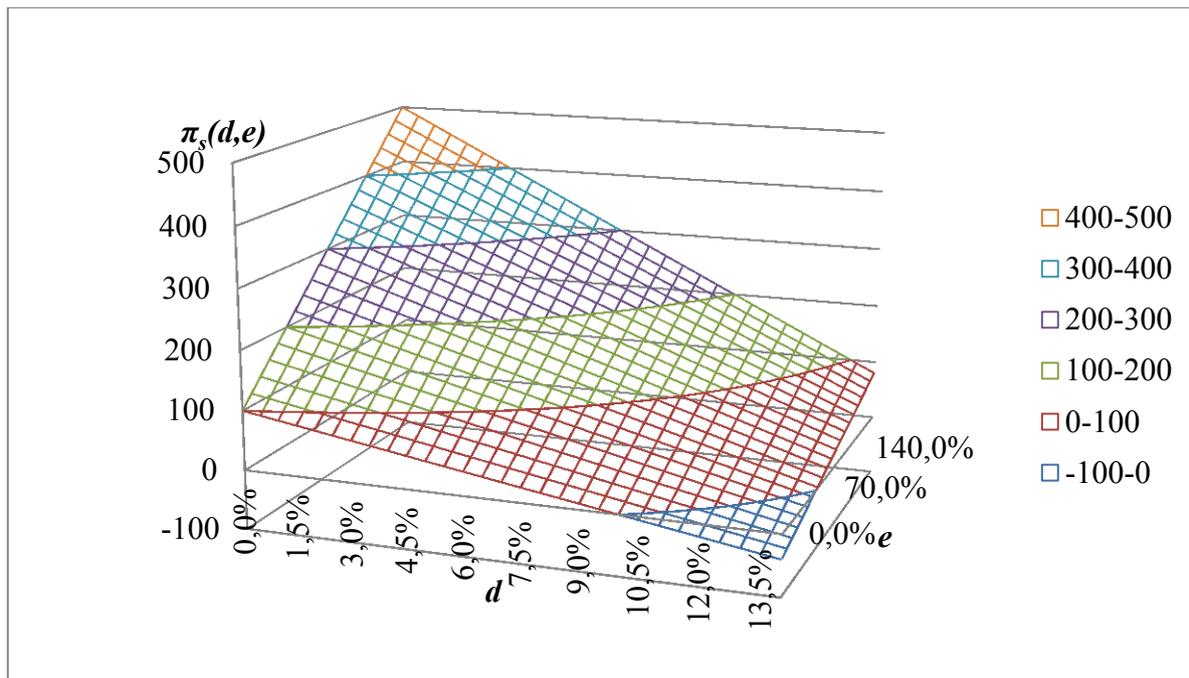
$$d > \frac{a - p e^2}{2p(1+e)}. \tag{15}$$

Assuming that the status quo point is the Bowley equilibrium, we can substitute right side of the equation (7) for p . This leads to the condition:

$$d > \frac{a - v e^2}{2(a+v)(1+e)}. \tag{16}$$

Hence, the minimum price discount for the buyer as the function of order growth rate is:

$$d_{min}(e) = \frac{a - v e^2}{2(a+v)(1+e)}. \tag{17}$$



Graph 2. Profit of the seller

On the other hand, seller also expects the rise of his profit.

$$\pi_s(q, p, d, e) > \pi_s(q, p), \tag{18}$$

$$p(1-d)q(1+e) - vq(1+e) - f_s > pq - vq - f_s, \tag{19}$$

which holds for:

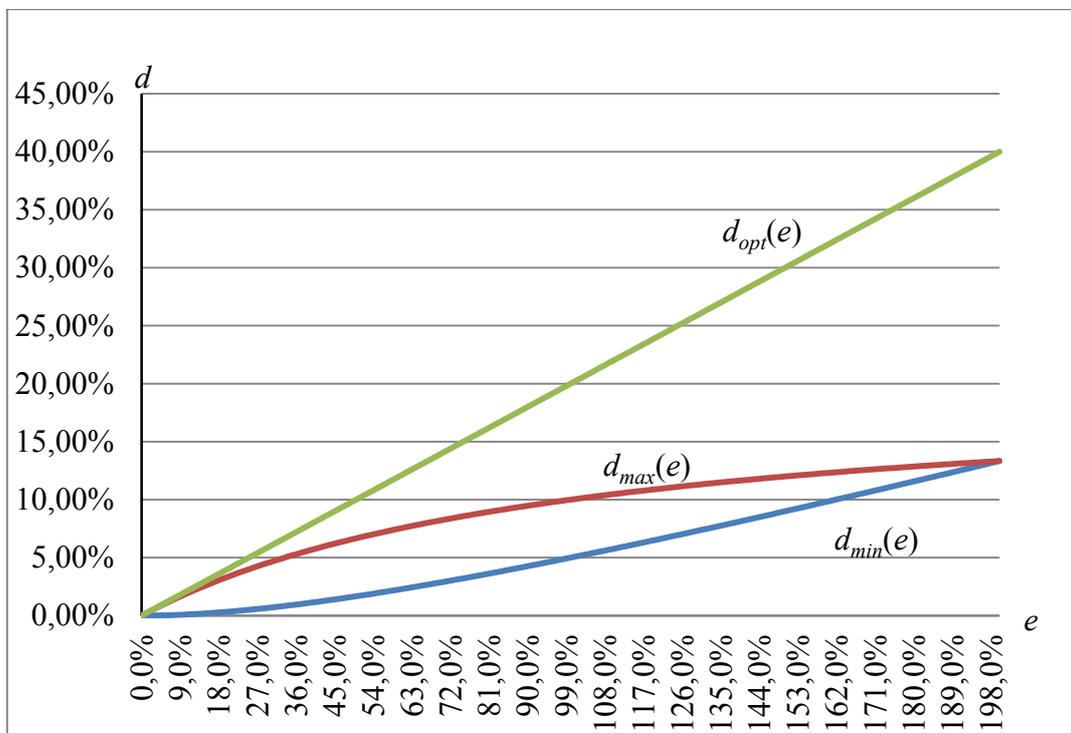
$$d < \frac{p-v}{p} \frac{e}{1+e}. \quad (20)$$

The condition shows, that maximum price discount of the seller depends positively on his margin rate and the seller's order growth rate. Assuming that the status quo point is the Bowley equilibrium, we can substitute right side of the equation (7) for p . This leads to the condition:

$$d < \frac{a-v}{(a+v)(1+e)} e. \quad (21)$$

Hence, the maximum price discount for the seller as the function of order growth rate is:

$$d_{max} e = \frac{a-v}{(a+v)(1+e)} e. \quad (22)$$



Graph 3. Relationships between e and d

The price discount agreement is possible if:

$$d_{max} e > d_{min} e \quad (23)$$

$$\frac{a-v}{(a+v)(1+e)} e > \frac{a-v}{2(a+v)(1+e)} e^2. \quad (24)$$

$a > v$ so, the condition (24) holds for:

$$0 < e < 2. \quad (25)$$

The chance for the price discount agreement bringing additional profits appears only if quantity growth rate is lower than 200%.

First, let's try to treat price discount for the order enlargement as the non-cooperative game. Buyer estimates the best possible answer for seller's discount in order to maximize his profit, which holds if the first derivative of the function (11) equals zero:

$$\pi'_b(e) = \frac{a(a-v)}{4b} - \frac{a-v}{8b} \frac{1+e}{1-d} - \frac{a^2-v^2}{8b} \frac{1-d}{1+e} = 0, \quad (26)$$

The equation leads to the formula for optimal relationship between d and e for the buyer:

$$e_{opt} d = \frac{a+v}{a-v} d. \quad (27)$$

The inverse function to the optimal quantity growth rate is:

$$d_{opt} e = \frac{a-v}{a+v} e. \quad (28)$$

Equation (28) shows optimal discounts maximizing buyer's profits at every given quantity growth rate. For every $e > 0$, $d_{opt} e > d_{max} e$. Optimal discount rate for the buyer can't be accepted by the seller. Hence, this solution can be rejected as non-cooperative option (see *Graph 3*¹). Best response for any buyer's quantity growth rate is the seller's price discount equal to zero. Best buyer's response for that call is zero quantity growth rate. Hence, Nash equilibrium (Nash, 1950b) for non-cooperative version of the game is the Bowley point ($e=0\%$, $d=0\%$).

2. Cooperative solution

According to Nash bargaining solution the set of the possible agreement points has to fulfil the Pareto optimality. The Pareto optimal set can be derived using the condition which equals Jacobi determinant for profit functions of the buyer and seller to zero:

$$J = \begin{vmatrix} \pi'_s(d) & \pi'_s(e) \\ \pi'_b(d) & \pi'_b(e) \end{vmatrix} = 0. \quad (29)$$

The derivatives of the profit functions are as follows:

$$\pi'_s d = -\frac{a^2-v^2}{8b} (1+e) \quad (30)$$

$$\pi'_s e = \frac{a(a-v)}{4b} - \frac{a-v}{8b} \frac{1+e}{1-d} - \frac{a^2-v^2}{8b} (1-d) \quad (31)$$

$$\pi'_b d = \frac{a^2-v^2}{8b} (1+e) \quad (32)$$

$$\pi'_b e = \frac{a^2-v^2}{8b} \frac{1-d}{1+e} - v \frac{a-v}{4b} \quad (33)$$

¹ Graphs and other calculations are made with the following assumptions $a=120$, $b=1$, $v=80$ and $f_b=f_s=100$.

Substituting equations 30-33 into the condition (29) gives:

$$-\frac{a^2-v^2}{8b}(1+e)\left[\frac{a^2-v^2}{8b}(1-d)-v\frac{a-v}{4b}\right]-\frac{a^2-v^2}{8b}(1+e)\left[\frac{a(a-v)}{4b}-\frac{(a-v)^2}{8b}(1+e)-\frac{a^2-v^2}{8b}(1-d)\right]=0 \quad (34)$$

Since $a > v$ and $e > 0$, the equation holds for:

$$e = 1. \quad (35)$$

The set of Pareto optimal solutions contains of all combinations of buyer's and seller's profits being the result of quantity growth by 100%. If the quantity traded is doubled in comparison to Bowley equilibrium, the profits fulfil the Pareto optimality criterion. This set is independent from the value of the price discount. Equation (35) substituted into the equations (17) and (22) leads to the formulas for minimum and maximum price discounts that cut the section of possible agreement in the Pareto optimal set:

$$d_{min} \ 1 = \frac{a-v}{4(a+v)}, \quad (36)$$

$$d_{max} \ 1 = \frac{a-v}{2(a+v)}. \quad (37)$$

Substituting the equations (35-37) into the profit functions, one can determine the borders of the negotiation set:

$$\pi_b \ d_{min}, \ 1 = \frac{(a-v)^2}{16b} - f_b, \quad (38)$$

$$\pi_s \ d_{min}, \ 1 = \frac{3(a-v)^2}{16b} - f_s, \quad (39)$$

$$\pi_b \ d_{max}, \ 1 = \frac{(a-v)^2}{8b} - f_b, \quad (40)$$

$$\pi_s \ d_{max}, \ 1 = \frac{(a-v)^2}{8b} - f_s. \quad (41)$$

This set of profits shows two interesting facts. First, in the Pareto optimal point with the minimal accepted by the buyer price discount gives the seller three times higher profit before fixed cost. Second, for maximal accepted by the seller price discount these profits are equal. If the fixed costs of the buyer and seller are equal, the second of them is in more convenient situation.

The Pareto optimal values of the minimum and maximum price discount for the assumed parameters are: $d_{min} \ 1 = 5\%$, $d_{max} \ 1 = 10\%$. This values lead to the following values of the profits: $\pi_b \ 5\%, 100\% = 0$, $\pi_s \ 5\%, 100\% = 200$, $\pi_b \ 10\%, 100\% = 100$, $\pi_s \ 10\%, 100\% = 100$.

According to Nash bargaining solution, the unique cooperative agreement is to fulfil the condition (Nash, 1950a, p. 159):

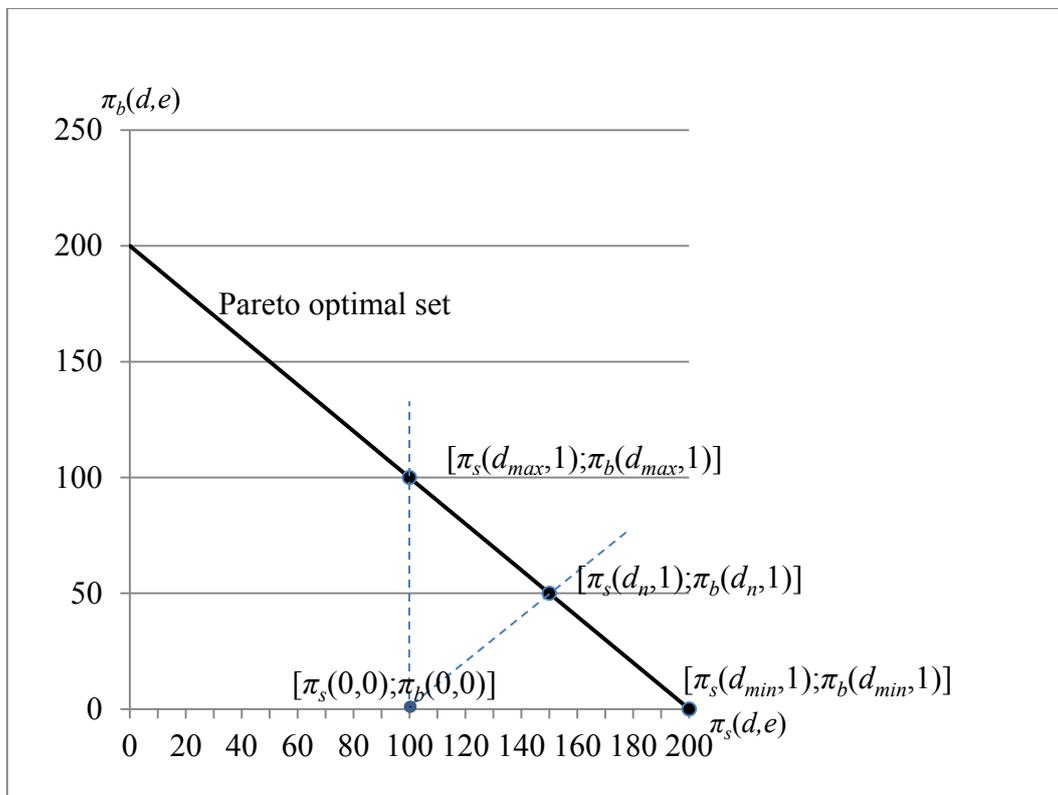
$$[\pi_b(d_n, 1) - \pi_b(0,0)][\pi_s(d_n, 1) - \pi_s(0,0)] = \max. \quad (42)$$

where:

d_n – price discount according to the Nash cooperative solution.

Left side of the equation 42 is called “Nash product”. It’s the product of the profit surpluses over the status quo point for both enterprises. The status quo point is the pair of profits without cooperation ($d=0$ and $e=0$). The Nash cooperative solution is the unique point (d_n) in the Pareto optimal set ($e=1$), which maximizes the Nash product. Substituting the profit functions (11) and (12) with the Pareto set condition ($e=1$) and the status quo point conditions ($d=0$ and $e=0$) into the equation (42), one can obtain:

$$\left[\frac{a(a-v)}{2b} - \frac{(a-v)^2}{4b} - \frac{a^2-v^2}{4b} (1 - d_n) - \frac{a(a-v)}{4b} + \frac{(a-v)^2}{16b} + \frac{a^2-v^2}{8b} \right] \left[\frac{a^2-v^2}{4b} (1 - d_n) - \frac{v(a-v)}{2b} - \frac{a^2-v^2}{8b} + \frac{v(a-v)}{4b} \right] = \max. \quad (43)$$



Graph 4. Nash bargaining solution

The necessary condition of the maximization is the first derivative of the Nash product, as the function of price discount, equal to zero:

$$\frac{a^2-v^2}{4b} \left[\frac{a^2-v^2}{4b} (1 - d_n) - \frac{v(a-v)}{2b} - \frac{a^2-v^2}{8b} + \frac{v(a-v)}{4b} \right] - \frac{a^2-v^2}{4b} \left[\frac{a(a-v)}{2b} - \frac{(a-v)^2}{4b} - \frac{a^2-v^2}{4b} (1 - d_n) - \frac{a(a-v)}{4b} + \frac{(a-v)^2}{16b} + \frac{a^2-v^2}{8b} \right] = 0, \quad (44)$$

which holds for:

$$d_n = \frac{3 a-v}{8(a+v)}. \quad (45)$$

The level of d_n is lower than d_{max} (formula 37) and higher than d_{min} (formula 36). The value of price discount according to Nash bargaining solution for the assumed parameters of the model is $d_n = 7,5\%$.

The derivation of the cooperative solution with alternative bargaining schemes leads to the same outcome. Kalai – Smorodinsky bargaining solution indicates the same point within the Pareto optimal set (Kalai, Smorodinsky, 1975). Egalitarian solution, which makes the additional profits over the status quo point equal, also points the same level of the price discount (Kalai, 1977). This coincidence occurs always for the linear Pareto optimal set.

Summary

The purpose of the article was to check whether the two stage cooperation between the seller and buyer within the bilateral monopoly produces the same solution as the agreement undertaken without price discount. Two stage cooperation is the agreement upon the discount on the price from the Bowley point and the quantity enlargement in comparison to this equilibrium. The alternative is simple cooperative solution derived directly for the price and quantity traded. In this situation status quo point is the option of no trade bringing losses equal to fixed costs. The cooperation with price discounts takes the Bowley equilibrium as the status quo point. The failure of negotiations leaves there both, the seller and the buyer.

Table 1. The comparison of two modes of cooperation

	Bowley point	One stage without price discount ²		Two stage with price discount	
		status quo point	Nash bargaining solution	status quo point	Nash bargaining solution
General solution					
p	$\frac{a+v}{2}$	-	$\frac{a+3v}{4b}$	$\frac{a+v}{2}$	$\frac{5a+11v}{16}$
q	$\frac{a-v}{4b}$	0,0	$\frac{a-v}{2b}$	$\frac{a-v}{4b}$	$\frac{a-v}{2b}$
d	0,0%	0,0%	0,0%	0,0%	$\frac{3}{8} \frac{a-v}{a+v}$
e	0,0%	0,0%	0,0%	0,0%	100,0%
Numerical example					
p	100,0	-	90,0	100,0	92,5
q	10,0	0,0	20,0	10,0	20,0
d	0,0%	0,0%	0,0%	0,0%	7,5%
e	0,0%	0,0%	0,0%	0,0%	100,0%
$\pi_s(d,e)$	100	-100	100	100	150
$\pi_b(d,e)$	0	-100	100	0	50

The result is higher profit of the seller, sharing the same sum of profits. He exploits his favorable strategic position of the price leader. The price leadership of the seller, establishing the pair of profits within the Bowley point, influences the division of profits within the cooperative Nash bargaining solution with price discount for quantity enlargement.

² The Nash bargaining solution implemented in the article to indicate price discount and the quantity growth rate, was also used to find the cooperative pair of the price and quantity traded. The equations (29) and (42) in this case were built for the profit functions $\pi_s p, q$ and $\pi_b p, q$.

One should add, that alternative two stage cooperation doesn't change the situation of the consumers. The market price, quantity and the consumer surplus are the same nevertheless which mode of cooperation is taken.

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